JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 99, NO. B9, PAGES 18,021–18,033, SEPTEMBER 10, 1994

Chaos in geomagnetic reversal records: A comparison between Earth's magnetic field data and model disk dynamo data

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Abstract. The Earth's geomagnetic field reverses its polarity at irregular time intervals. However, it is not clear whether a reversal is a deterministic (low-dimensional) or a random (high-dimensional) process; the duration-frequency distribution of the polarity time intervals resembles those generated by random processes, but many models suggest that a geomagnetic field reversal can be the outcome of a deterministic dynamics, that of the convection in the Earth's outer core. The latter, in turn, is only a part of an extremely complex system, made up of both terrestrial and extraterrestrial subsystems nonlinearly interacting with each other over a wide range of time scales. We studied the geomagnetic field reversal patterns by means of several techniques of nonlinear dynamics and compared the results obtained on actual geomagnetic reversal data with synthetic reversal sequences generated by the Rikitake and Chillingworth-Holmes models of the Earth's magnetic field. We analyzed both the geomagnetic and the synthetic reversal scales by nonlinear forecasting and found that we cannot predict the geomagnetic reversal sequence with nonlinear forecasting. Predictability of the synthetic data varies widely depending on the model parameters. Phase portraits of data obtained from the magnetic field models show fractal structures similar to those associated with the Lorenz attractor. We measured the correlation dimension D_C of the synthetic and geomagnetic data by means of the Grassberger-Procaccia method and found that D_C always has a value of about one for the synthetic data. The correlation integrals for the geomagnetic reversal sequence behave very differently from those of randomized reversal sequences and suggest that the Earth's geomagnetic field reversal dynamics is not random. However, the limited size of the magnetic reversal data set (282 points) and the poor convergence of the correlation integrals make a quantitative assessment of low-dimensional chaos impossible. Our analysis sets a lower limit to the correlation dimension of the geomagnetic reversal dynamics: $D_C > 3$.

Introduction

The temporal pattern of reversals of the Earth's magnetic field has been extensively studied, and beginning with *Rikitake* [1958], it has been modeled by self-excited bistable disk dynamos that show auto-induced reversals. These models are simplified descriptions of convection in the Earth's outer core, which is believed to be an electrically conductive fluid where the dipole component of the Earth's magnetic field is originated. The dynamics of the Rikitake dynamo, as well as those of slightly different dynamo models [*Robbins*, 1976; *Chillingworth and Holmes*, 1980; *Shimizu and Honkura*, 1985; *Rasband*, 1990], have all been shown to be chaotic [*Cook and Roberts*, 1970; *Ito*, 1980; *Chillingworth and Holmes*, 1980; *Bergé et al.*, 1984; *Shimizu and Honkura*, 1985; *Holden and Muhamad*, 1986; *Hoshi and Kono*, 1988; *Rasband*, 1990]; *Tritton* [1989] put forward the idea that the Earth's magnetic field reversal dynamics can be modeled as that of an excited nonlinear pendulum. In such models the polarity reversal is driven by a low-dimensional deterministic dynamics and is therefore not a random process.

Because the frequency distribution of the temporal duration of the geomagnetic periods has been interpreted to be a Poisson or a gamma distribution [see *Marzocchi and Mulargia*, 1990, 1992], a geomagnetic reversal is often thought to be a random (high-dimensional) process, triggered by any one or a combination of a number of possible external disturbances [e.g., *Lutz and Watson*, 1988; *Lowrie*, 1989], including meteoritic impacts [*Glass and Zwart*, 1979; *Glass et al.*, 1979]. Thus there is an unresolved contradiction between the output of the deterministic models and the random character of the geomagnetic reversals often implied in studies of the size-frequency distribution of such data. Although interesting attempts were made to model the Earth's geomagnetic reversals as a deterministic process with noise [*Crossley et al.*, 1986], those who study the behavior of models tend to think of the geomagnetic reversals as deterministic, and those who study the Earth's magnetic field itself tend to think of its reversals as a stochastic/random process.

Many different interactions are described in the literature between the Earth's magnetic field dynamics and a number

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Paper number 94JB01237.

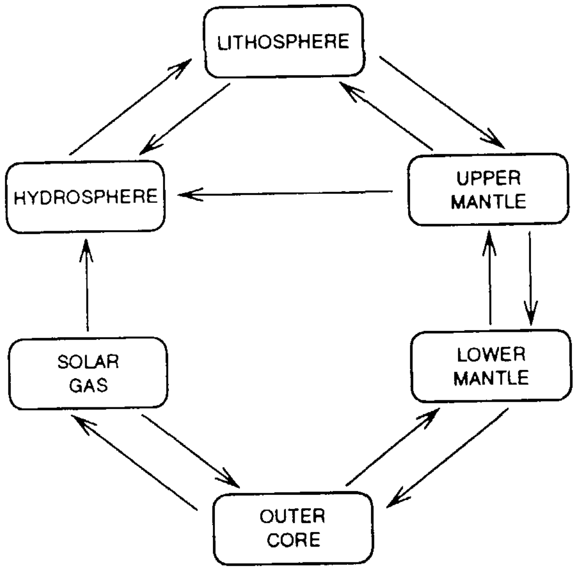


Figure 1. Diagram illustrating schematically how the Earth's magnetic dynamo couples nonlinearly with many different terrestrial and extraterrestrial systems. Each arrow represents one interaction documented in the literature. Interactions may be different at different timescales. Some systems, such as the solar gas, the mantle, and the atmosphere (a subsystem of the hydrosphere) are thought to have a chaotic dynamics.

of different systems, which can act as external forcing factors. Figure 1 is an attempt to summarize these interactions in a coherent picture. Convection in the outer core is probably coupled to convection in the lower mantle [*McFadden and Merrill*, 1984; *Merrill and McFadden*, 1990; *Laj et al.*, 1991], which is thought to be chaotic [*Stewart and Turcotte*, 1989]. *Vogt* [1975] suggested that a reversal rate change took place at the time of a major rearrangement of the lithospheric plate migration pattern and that the core and the upper mantle dynamics must be coupled in some way. Variations in the geomagnetic field may correlate with some hydrosphere circulation changes [*Morner*, 1989] because the fluid outer core and the hydrosphere can interchange angular momentum [e.g., *Rochester*, 1984]. Further nonlinear interactions can be derived from the influence of the Sun's activity on the Earth's climate [*Ribes et al.*, 1987; *Friis-Christensen and Lassen*, 1991], which can influence the core's dynamics via interchange of angular momentum between the hydrosphere and the outer core.

These observations favor a chaotic model for the Earth's magnetic field dynamics. In fact, sensitivity to minute external forces (and nonlinear interactions with a large number of processes, giving rise to feedback) is typical of natural systems in a chaotic regime. Thus a new picture is beginning to emerge, in which the geomagnetic dynamics appears to be part of an extremely complex system, where several terrestrial subsystems (core, mantle, lithosphere, and hydrosphere) are coupled and tuned to each other and to extraterrestrial dynamics as well (Figure 1).

The main question we address is: Do all the "external influences" (i.e., the coupling of core dynamics with subsystems other than the outer core) mask or erase an underlying low-dimensional deterministic geomagnetic dynamics related to outer core convection; or is there a recognizable determinism under some amount of noise? This question can be addressed by measuring the correlation dimension of the dynamics. If the dimension of the geomagnetic field dynamics is relatively low, then the system appears to be deterministic and low dimensional. If the correlation dimension does not converge to a low value (usually less than 10), then the system is either high dimensional (that is, it is governed by such a high number of parameters that its dynamics cannot be distinguished from a random one), or noise masks any underlying low-dimensional determinism that may exist. However, the distinction between these possibilities is a very arbitrary matter.

As explained later, given the small data sets used, we cannot distinguish a system with a correlation dimension larger than about three from a truly high-dimensional system. Thus because of the many variables that affect the Earth's magnetic field, some of which are discussed above and represented in Figure 1, it may appear hopeless to find anything other than randomness and noise. However, there are many remarkable examples of complex natural systems that exhibit self-organization and order to a great degree, including the Darwinian evolution of life [*Eigen and Schuster*, 1979; *Prigogine and Stengers*, 1984; *Dawkins*, 1987]. A well-studied example is that of convection in the Rayleigh-Bénard configuration [*Nicolis and Prigogine*, 1977; *Bergé et al.*, 1984]. When the Rayleigh number Ra is lower than a critical value Ra_c there is no convection; the dynamics of the molecules is due only to thermal motion, and the number of state variables is of the order of Avogadro's number. When Ra exceeds Ra_c convection begins, and the collective motion of the fluid can be described by as little as two or three state variables; the system has undergone self-organization.

Well-known examples of highly self-organized systems in geological sciences include the spectacular mineral layering structures found in many fossil magma chambers [*Wager and Brown*, 1968; *McBirney and Noyes*, 1979; *Boudreau*, 1984; *Merino*, 1984], which appear highly organized, although magma chambers are very complex systems. Also, the dynamics of an active magma chamber may be low dimensional [*Cortini et al.*, 1991; *Cortini and Barton*, 1993]. Thus it is not unreasonable to look for a deterministic structure in the geomagnetic record, in spite of the complexity of the system.

We compare the pattern of the Earth's magnetic reversals with synthetic magnetic reversal records generated by the disk dynamo model of *Rikitake* [1958] and *Chillingworth and Holmes* [1980] and look for determinism using three techniques of nonlinear dynamical systems (phase portraits, nonlinear forecasting, and the Grassberger-Procaccia method); the latter two will be outlined in the following section.

Dubois and Pambrun [1990] analyzed the geomagnetic reversal sequence with the Grassberger-Procaccia algorithm and a sliding window technique, and they reported a value of the correlation dimension $D_C \sim 2$ for the period between 140 and 23 m.y.; they found higher values of D_C for the period between 23 and 0 m.y. They suggested that a quantitatively significant value of D_C can be obtained for data sets of only 50–100 points, and as explained below, we disagree on this point.

Two Techniques of Chaos

The mathematical theory of chaos, or dynamics of nonlinear systems, can be very useful in the study of complex systems. In favorable cases it can distinguish systems whose macroscopic behavior and evolution is dominated by high-dimensional noise or random processes from those whose behavior and evolution is deterministic and controlled by a relatively low-dimensional dynamics, although the dynamics may never be known. Introductions to chaos and chaotic attractors are given by *Cvitanovic* [1984], *Bergé et al.* [1984], *Thompson and Stewart* [1986], and *Schaffer et al.* [1988]; a more formal treatment is given by *Sagdeev et al.* [1988], and an excellent popular book is that of *Gleick* [1988]. In this section we present the two techniques of chaos that we apply in this study: nonlinear forecasting and the so-called Grassberger-Proccacia method.

The forecasting method we use in this paper is described by *Sugihara and May* [1990] and is implemented by Dynamical Systems, Incorporated [*Schaffer and Tidd*, 1990]. In nonlinear forecasting the time series is first lagged and embedded in a n-dimensional phase space [*Takens*, 1981]. A mapping of the visited phase space volume onto itself is then constructed in the following way. A small phase space volume dV_n, which is visited by the system at time t_n, is associated to a new volume dV_{n+1}, visited by the system at time $t_n + dt$. This procedure is repeated for every point visited by the system. This mapping of the phase space volume can now be used to formulate predictions about the future evolution of the system. The difference between a stochastic dynamics and a deterministic chaotic dynamics can now be described in the following way. A random dynamics, which is perturbed by myriads of different causes, from a small phase space volume dV_n can move to any nearby volume. A chaotic dynamics, on the contrary, because it is deterministic, is forced to move in a well-defined direction. When predicting the future state of data that occupy a phase space volume dV_n, because of the sensitivity to initial conditions, dV_{n+1} will be larger than dV_n, and for a k large enough, dV_{n+k} will cover the entire visited volume. This method allows predictions, which can be very useful for $k < k_0$, k_0 being some threshold value. It is empirical in that the best value for the time lag, the embedding dimension, and other parameters are adjusted by trial and error. The best values are those that minimize the errors on the predictions.

One key difference between deterministic chaos and a random dynamics is that a chaotic dynamics may be controlled by a relatively small number of state variables (fewer than 10). If, on the contrary, the dynamics depends on microscopic perturbations that are amplified, then the number of state variables is very large, and the dynamics is indistinguishable from a random one. This difference provides the basis for identifying a chaotic process by means of the so-called Grassberger-Procaccia method. This is one way to evaluate the correlation dimension of an attractor, which is related to the fractal dimension [*Grassberger and Procaccia*, 1983]. In this method one embeds the attractor in an n-dimensional space, with increasing n; that is, one plots a trajectory in a space $y(t + T)$ versus $y(t + 2T)$ versus $\cdots y[t + (n - 1)T]$. If the fractal correlation dimension D_C of the attractor continues to grow with n, this suggests that the dynamics is not low dimensional and that the failure of D_C to converge results from either high dimensionality or amplification of noise, that is, of random perturbations [*Grassberger and Procaccia*, 1983; *Grassberger*, 1986; *Schaffer et al.*, 1988; *Lorenz*, 1991]. If the correlation dimension D_C of a nonperiodic dynamics converges to a low value, then the dynamics may be deterministic chaos, governed by a small number of state variables (which is the minimum value n of the embedding dimension D_e for which D_C converges), and its long-term unpredictability derives from its attractor being chaotic.

There has been considerable discussion regarding the validity of this approach, especially on the minimum size of a data set needed for a measurement of dimensionality to be reliable [e.g., *Grassberger*, 1986; *Schaffer et al.*, 1988; *Tsonis and Elsner*, 1990]. In our experience, small sets of data (a few hundred points) do not allow a reliable measure of the exact value of the correlation dimension (D_C). Errors may derive from a variety of sources but, in particular, from the small size of the data set. *Eckmann and Ruelle* [1992] showed that if a data set contains N points, then the value of D_C obtained by the Grassberger-Procaccia method cannot exceed $2 \log_{10} N$, which, for 282 data points, is about 4.9. However, this is a theoretical limit for high-precision data. A realistic limit for noisy data such as the geomagnetic reversal set is lower than 4.9, probably around three. Therefore we consider the actual values we calculated for D_C to be unreliable. However, our results suggest that the Grassberger-Procaccia method can distinguish qualitatively between a data set of 282 points whose D_C converges to a relatively low value and one where D_C does not converge. We analyzed a sequence of 282 points selected randomly from a data set of 10,000 points of known D_C (Mackey-Glass equation for time delay of 30 s, where $D_C \sim 3.04$ [see *Schaffer et al.*, 1988, p. 1.73]). The Grassberger-Procaccia method converged to $D_C \sim 2$, quantitatively too low but qualitatively converging to low-dimensional chaos. We then randomized the 282 points; the method did not converge to any value of D_C (the figure was similar to Figure 10c), thereby implying no recognizable structure in the data.

Timescale and Nonlinear Forecasting Analysis of the Earth's Geomagnetic Field Reversals

On the basis of the analysis of many marine magnetic profiles, *Cande and Kent* [1992] defined a new magnetic field reversal timescale from the Late Cretaceous to the present. Because the scale of *Cande and Kent* [1992] does not go back farther than 83 m.y., we have merged it with the portion of the scale for the time interval 83–160 m.y. [*Kent and Gradstein*, 1986] in order to analyze a timescale from the present to 160 m.y. ago. For both the real and the synthetic reversal scales we have studied the sequence of the "magnetic periods" $X_n = t_{n+1} - t_n$.

The first question we asked is whether the duration X_n of the nth magnetic interval is determined by the length of the preceding intervals $X_{n-1}, X_{n-2}, \cdots$ This seemed a reasonable hypothesis, as in the dynamo models of *Rikitake* [1958] and *Chillingworth and Holmes* [1980], the dynamics of the magnetic field is strictly deterministic; moreover, there are systems, such as the dripping faucet of *Shaw* [1984], whose attractors are determined by discrete time intervals (those between falling drops), which might resemble those between magnetic reversals. In order to answer this question we employed the nonlinear forecasting method [*Farmer and Sidorowich*, 1988; *Casdagli*, 1989; *Sugihara and May*, 1990], as implemented by Dynamical Systems, Incorporated, whose programs we used for many of the numerical methods

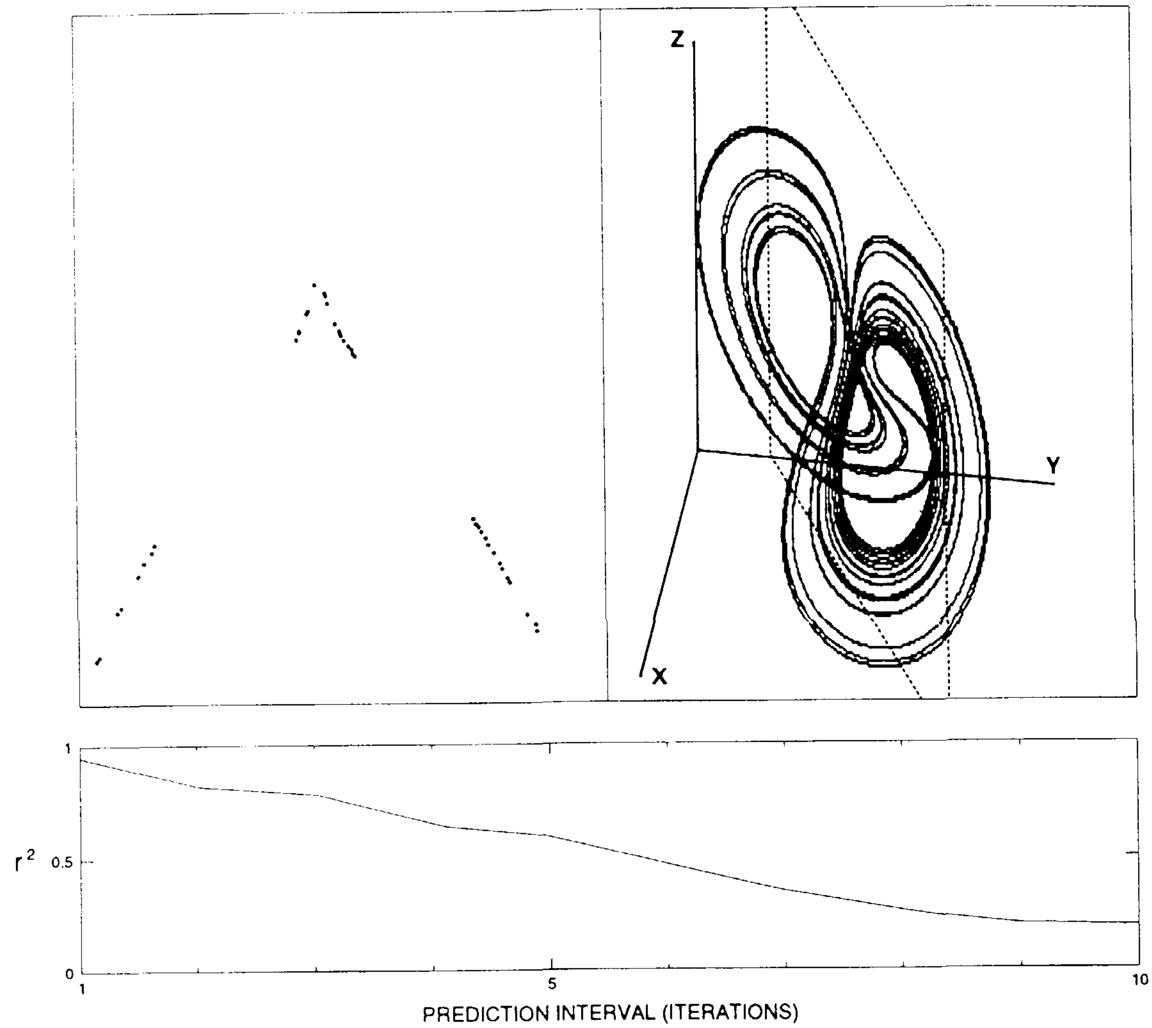


Figure 2. (top right) Phase portrait of the attractor of the disk dynamo model by *Rikitake* [1958], described by (1), for $\mu = 1$ and $k = 1.5$. The attractor is cut by a vertical Poincaré plane (shown), and the corresponding Poincaré section is shown on the top left. (bottom) Results of nonlinear forecasting applied to the same Poincaré section of 350 points obtained by cutting the disk dynamo attractor with a Poincaré plane (top right). Here r^2 is the linear correlation coefficient between the predicted values and the observed values for each particular prediction interval. Nonlinear forecasting can predict the position of the next point to a good degree of accuracy (correlation coefficient $r^2 \sim 0.91$ for prediction interval $dt = 1$). As is typical of chaotic flows, the predictability (r^2) decreases as the forecasting interval increases.

described in this paper [*Schaffer et al.*, 1988; *Schaffer and Tidd*, 1990]. For nonlinear forecasting we always set the embedding dimension $D_e = 2$, the embedding lag $T = 1$, and the prediction epsilon (which controls the size of the phase space volume dV_n) $\epsilon = 0.005$. The results, however, do not critically depend on the choice of the parameter values.

The sequence of 282 points that constitutes the geomagnetic reversal timescale appears unpredictable by nonlinear forecasting; again, different choices of parameters do not modify this result. This result does not rule out the possibility of Earth's magnetic field dynamics being chaotic. We will show below that synthetic magnetic reversal sequences from chaotic, low-dimensional, deterministic models can be unpredictable too.

Nonlinear Analysis of Dynamo Models

We have generated synthetic magnetic reversal sequences from two different disk dynamo models in order to compare them with the geomagnetic intervals. The first dynamo model [*Rikitake*, 1958; *Ito*, 1980] is described by the following equations:

$$dx/dt = -\mu x + zy$$

$$dy/dt = -\mu y + z'x \qquad (1)$$

$$dz/dt = dz'/dt = 1 - xy$$

From the third expression one gets $z' = z + a$, where $-a = \mu(K^2 - K^{-2})$; the Rikitake system has two fixed points, whose coordinates in phase space are $x = \pm K$, $y = \pm K^{-1}$, and $z = \mu K^2$. Here μ^2 is the ratio between the "mechanical timescale" of the system (the time that the discs would take to accelerate to the typical angular velocity in the absence of Lorentz forces) and the "electromagnetic diffusion time"; that is, the time constant by which the field would decay if the discs were stopped [*Cook and Roberts*,

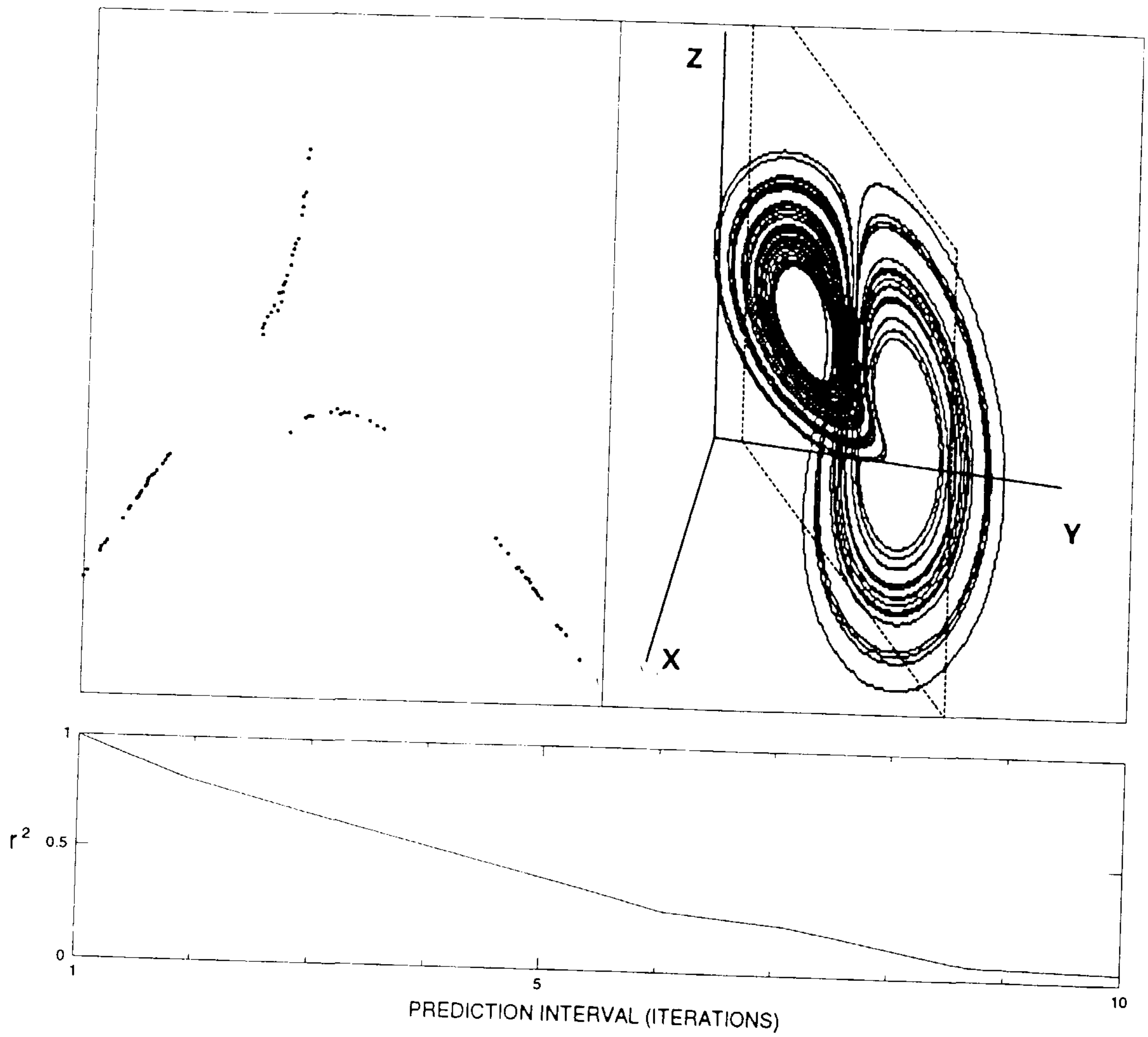


Figure 3. (top right) Phase portrait of the dynamo model of *Chillingworth and Holmes* [1980], described by (2), with $a = 5.5$, $b = 14.625$, and $c = 1$. The attractor is cut by a Poincaré plane (shown), and the corresponding Poincaré section is shown on the top left. (bottom) Results of nonlinear forecasting applied to the same Poincaré section of 163 points obtained by cutting the disk dynamo attractor with a Poincaré plane (top right). In spite of the small number of analyzed points (163), for prediction interval $dt = 1$ we obtain $r^2 \sim 0.97$.

1970]. The latter time constant is believed to be, for the Earth's magnetic field, of the order of 10,000 years [*Merrill and McElhinny*, 1983].

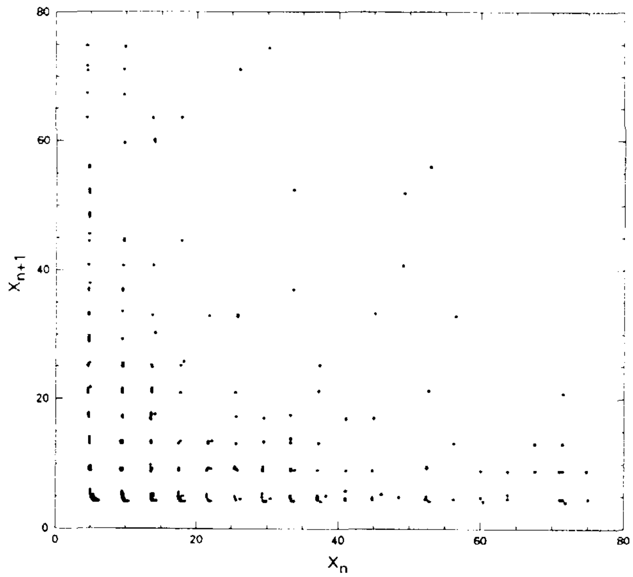
The second dynamo model [*Chillingworth and Holmes*, 1980] is described by

$$dx/dt = a(y - x)$$
$$dy/dt = zx - y \qquad (2)$$
$$dz/dt = b - xy - cz$$

Using a Runge-Kutta fourth-order routine, with $dt = 0.0005$, we have integrated (1) and (2) for several different values of the constants, always in a chaotic regime, and have obtained reversal records 600 to 20,000 points long. Integration of (1) and (2) yields attractors (Figures 2 and 3) that are similar to the Lorenz attractor [*Lorenz*, 1963]; moreover, for both dynamo models, successive maxima z_n of the z variable exhibit a cusplike structure on a z_n, z_{n+1} plot (time one return map) similar to that of the Lorenz attractor [*Lorenz*, 1963]. Stimulated by these similarities between the Lorenz attractor and those of the dynamo systems, we have also performed some integrations of the Lorenz system as described below.

We arbitrarily took the model magnetic field to be normal when the value of x is positive and reversed when x is negative. Thus each time x changed its sign we assumed that a magnetic reversal took place and recorded the time t_n. We analyzed the synthetic sequence of "magnetic periods" $X_n = t_{n+1} - t_n$; in this way we produced a synthetic reversal record that can be compared with the actual Earth's magnetic field reversal timescale.

The results we obtained by analyzing the synthetic reversal records by nonlinear forecasting vary wildly as a function of the parameters. For both models (and for the Lorenz system as well) the linear correlation coefficients r^2 between the observed and the predicted values at one time interval into the future can vary between almost unity and almost zero, depending on the choice of the parameters, although all



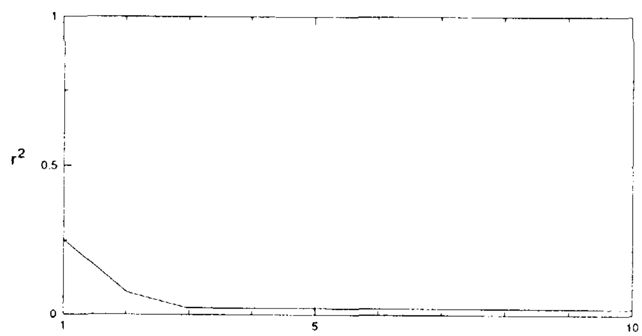


Figure 4. (a) Two-dimensional time lag phase portrait for one synthetic reversal record X_n of 731 points (Rikitake dynamo, equation (1), with $\mu = 1$ and $k = 1.5$). The "magnetic interval" duration X_{n+1} is plotted as a function of X_n. (b) Results of nonlinear forecasting on the sequence X_n (731 points), which appears to be almost completely unpredictable.

the sequences are obtained by deterministic equations (see Figures 4 to 7). Of course, the predictability of a synthetic record also depends heavily on its length (see Figure 7b). This is because the wider the mapping of the visited phase space, the more accurate the predictions can be. If a chaotic data series 10,000 points long, for which nonlinear forecasting gives $r^2 \sim 0.8$, is truncated to 282 points (the number of actual geomagnetic field reversals), r^2 then drops close to zero. Thus the negative result we obtain by analyzing the geomagnetic reversal scale by nonlinear forecasting ($r^2 \sim 0$, apparent complete unpredictability) does not rule out the hypothesis of low-dimensional chaos in the geomagnetic data.

Nonlinear forecasting works consistently well in predicting a different set of data; that is, the sequences of points obtained by cutting both the Lorenz and the dynamo attractors with a Poincaré plane (Figures 2 and 3). Predictions of Poincaré sections are good even for the parameter values that yield the most unpredictable reversal sequences and for small data sets of only 150–300 points. *Sornette et al.* [1991, p. 11,937] analyzed the sequence of repose periods between eruptions in two volcanoes (data similar in nature to our magnetic interval sequences) and wrote that "In the case of the attractor constructed from the sequence of repose periods between volcanic eruptions, the analog of a Poincaré section is the extraction of a sub-sequence from the full sequence of repose periods between eruptions." We disagree on this point. In fact, a sequence of points of a Poincaré section contains more information than our studied sequences X_n. In our data series, t_n represents the time when one state variable assumes a particular value ($x(t) = 0$; see equations (1) and (2)). Therefore each of our data points is characterized by two scalar components: t_n and $x(t_n) = 0$. In contrast, a Poincaré section of a three-dimensional system consists of a series of points in a three-dimensional phase space, and these are vector quantities, each one being characterized by three components: $x(t_n)$, $y(t_n)$, and $z(t_n)$, the state variables of the system. To appreciate this difference consider the following example. Suppose one has, at regular space intervals, the position and speed of a car driving on a highway (points on a Poincaré section); clearly, one has more information than if the only known parameter were the time when the car crosses the line between two contiguous lanes (sequence X_n). Finally, we find the conclusions of *Sornette et al.* [1991] are not supported by their analysis mainly because they arbitrarily removed data points that did not fit their one-dimensional map. Even assuming low-dimensional chaos in their data, there is no reason to believe that their phase portrait should fit a one-dimensional map and not a more complex one like the Hénon attractor or like those we present in Figures 4a, 5a, 6a, and 7a.

When the synthetic "magnetic interval" duration X_{n+1} is plotted as a function of X_n, we obtain the phase portraits reported in Figures 4a, 5a, 6a, and 7a. These figures are chosen to represent some of the typical appearances of the many phase portraits we obtained; in particular, the plot in Figure 4a is very similar to those obtained from the Rikitake model with the parameter values ($k = 2$, $\mu = 1$ to 2) suggested by *Cook and Roberts* [1970]. These phase portraits vary greatly in configuration and complexity as a function of the model parameters, but they seem to repeat a few basic patterns that are clearly visible in Figure 5a, and these clearly seem to have a fractal structure (see Figures 5a–5c).

The variability of the phase portraits as a function of the model parameters causes the variable results produced by nonlinear forecasting for the various synthetic magnetic records. Phase space is stretched and folded more rapidly as the attractors become more complex, and this makes prediction of their behavior more difficult. This effect can be quantified by measuring, with a technique described by *Wolf* [1986], the maximum Lyapunov exponent of the data series, which is defined as

$$\lambda_i = \lim_{t \to \infty} (1/t) \ln_2 (d_t / d_{t=0}) \tag{3}$$

and gives a measure of the divergence of two nearby orbits in phase space. In (3), t is time, $d_{t=0}$ is the distance between two points in phase space at time $t = 0$, and d_t is the distance between the same points at time t. A Lyapunov exponent $\lambda_i > 0$ indicates a chaotic dynamics. For our model reversal series the maximum Lyapunov exponents, calculated with the software of Dynamical Systems Incor-

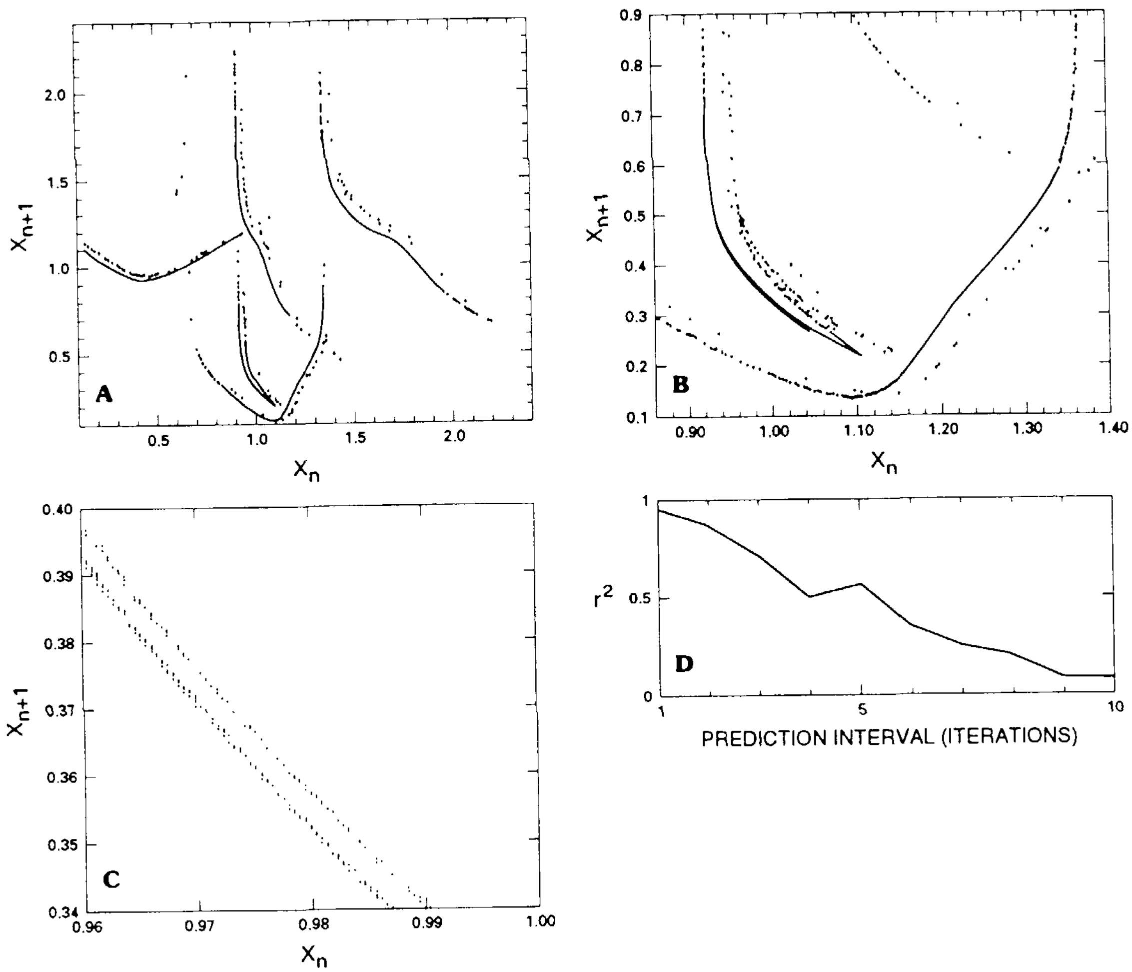


Figure 5. (a) Two-dimensional time lag phase portrait for one synthetic data sequence X_n of 9511 points (Rikitake dynamo, equation (1), with $\mu = 2$ and $k = 0.2$). (b) and (c) Sequential magnifications of a portion of Figure 5a are suggestive of a fractal structure. (d) Predictions of nonlinear forecasting for this simpler attractor ($r^2 \sim 0.90$ for a sequence reduced to 300 points at $dt = 1$) are much better than for that of Figure 4.

porated, range from about 0.6 for the data in Figure 5 to about 3.6 for the data in Figures 4 and 7. The units of λ_i are bits of information lost per time interval. The higher the λ_i, the more sensitive the dynamics is to initial conditions, and the more difficult it is to forecast.

The basic structure of these attractors looks like, and may be related to, the Hénon attractor [*Hénon*, 1976], which was constructed to model the behavior of the Lorenz attractor with a discontinuous system. The exact nature of this relationship, how these structures vary as a function of the parameters, and how they could be described by difference equations, are problems beyond the scope of this paper. It is clear, however, that the structures shown in Figures 4–7 behave as attractors. A single closed loop (limit cycle) of the Lorenz attractor, for example, corresponds to a single point in the X_n, X_{n+1} phase diagrams, and periodic cycles correspond to a finite number of points. Based on the aperiodicity of the synthetic polarity reversal records, and on visual inspection of Figures 5a–5c, we suggest that, in a chaotic regime, the X_n, X_{n+1} phase diagrams must really be fractal, although we have not been able to demonstrate this point.

When the geomagnetic reversal data are represented in a X_n, X_{n+1} plot (Figure 8), they appear featureless in comparison with the model attractors, although qualitatively the density distribution on the geomagnetic data is rather similar to those of some model sequences; the attractor in Figure 4, with some noise added, might look rather similar to the actual data, but we have not demonstrated this.

The reversal sequences X_n, generated by the two model dynamos can also be represented in a three-dimensional space. Figure 9a shows one of such sequences, where the points have been connected by straight lines. This figure appears to contain some structure. *Sornette et al.* [1991], in their analysis of the sequence of repose periods between volcanic eruptions, interpreted such a structure in their data as suggesting that the temporal sequence of volcanic eruptions is controlled by deterministic chaos. However, our model magnetic reversal scale, after randomization (in which any structure in the sequence of the magnetic periods is destroyed), appears to contain a degree of structure similar to the original (Figure 9b). Some randomized model data sequences, made by using different seed numbers, appear even more structured than the original sequence. Thus the structure perceived by the human eye in Figure 9 is only

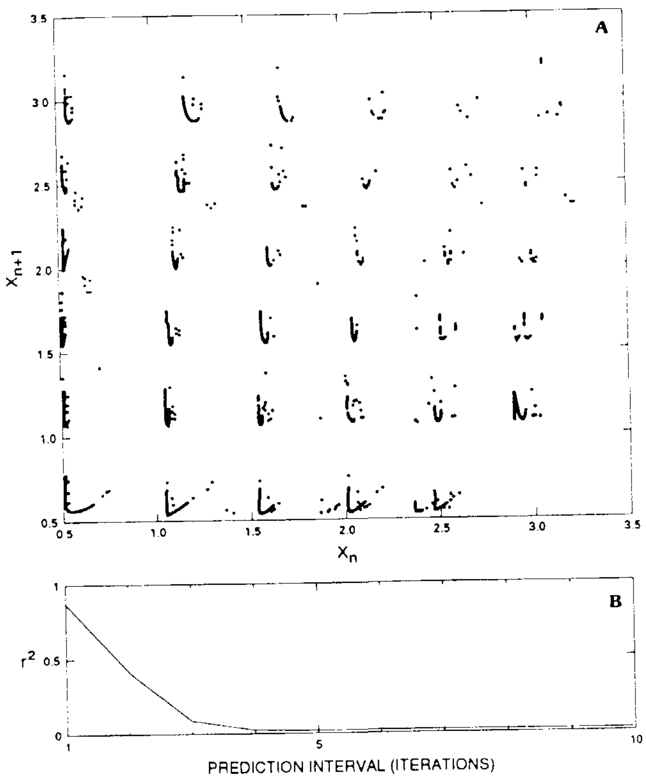


Figure 6. (a) Two-dimensional time lag phase portrait for one synthetic data sequence X_n of 4461 points obtained from the Lorenz system ($a = 10$, $b = 50$, and $c = 8/3$). The attractor in this figure has similarities to those in Figures 4 and 7. (b) Results of nonlinear forecasting on the sequence X_n.

apparent. We suggest that the pattern in these three-dimensional plots is even less distinct than it is in the two-dimensional plots, like those in Figures 4–8, which further suggests to us that the structure observed in these plots for volcanic repose times by *Sornette et al.* [1991] may also be only apparent.

Correlation Dimension of The Geomagnetic Reversal Record and Model Data

Another possible approach to analyzing magnetic records is the Grassberger-Procaccia method [*Grassberger and Procaccia*, 1983; *Grassberger*, 1986; *Lorenz*, 1991]. The concept of this approach is that the dimension of a noisy, or random, dynamics tends to infinity; in contrast, a deterministic chaotic dynamics may be of low dimension. Thus if one has an aperiodic dynamics, and can demonstrate that the fractal dimension of the attractor is low (usually less then 10), this is taken to mean that the dynamics is deterministically chaotic.

We used the Grassberger-Procaccia method to analyze the synthetic records for both the dynamo models and for the Lorenz model; we further analyzed the actual geomagnetic reversal sequence. The value of the correlation dimension (D_C) we obtained for the dynamo models and the Lorenz model is always approximately one (Figure 10a). This result is quantitatively reliable because (1) we consistently obtain it for all the model record sequences, and (2) the model records are much longer than the geomagnetic records (from 600 up to about 20,000 points). *Grassberger and Procaccia* [1983, p. 191] reported that $D_C \leq D$, the latter being the fractal or Hausdorff-Besikovich dimension, and that "in general the inequality is rather tight." Thus the fractal dimension of the attractors depicted in Figures 4–7 is approximately one. One feature of Figure 10a is that the slopes of the correlation integrals seem to increase indefinitely for values of $\ln(g)$ (where g is the correlation length, normalized to the overall attractor length) higher than about -6 but converge for values between about -11 and -7. The reason is that at large values of g one is looking at the distribution of the basic patterns on the plane (see Figure 4a), and that is not fractal. It is the structure within those patterns that has a low value of the fractal dimension, as is

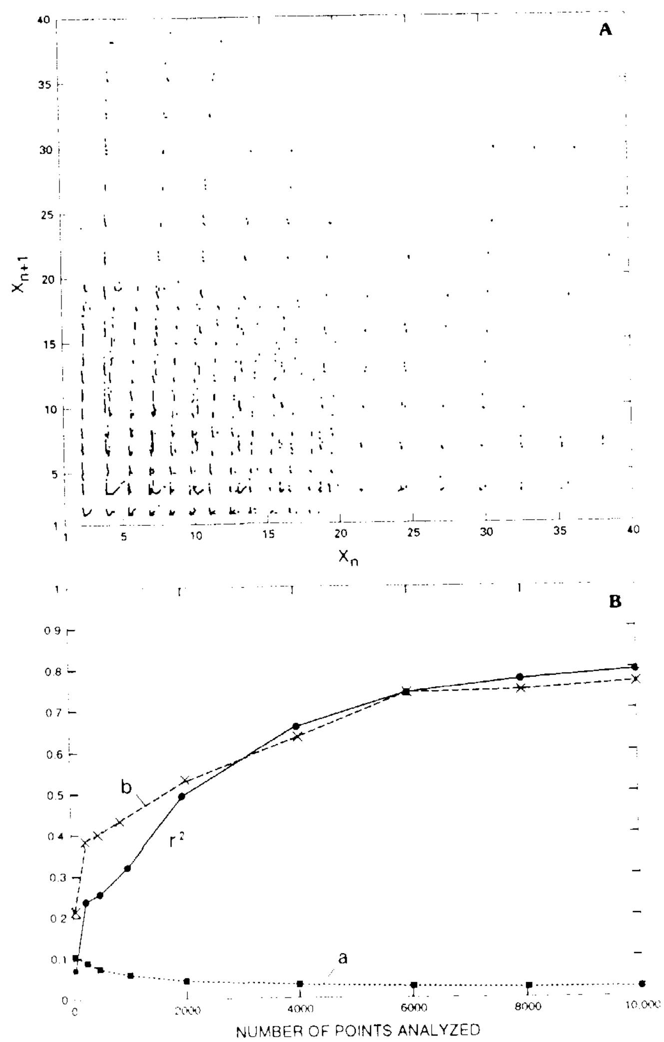


Figure 7. (a) Two-dimensional time lag phase portrait for one synthetic record X_n of 19,083 points (Chillingworth and Holmes dynamo, equation (2), with $a = 5.5$, $b = 14.625$, and $c = 1$). (b) Results of nonlinear forecasting applied to the same sequence, for $dt = 1$, as a function of the number of points analyzed. Curves a and b are the parameters of the regression line $y = bx + a$; r^2 is the linear correlation coefficient. For perfect predictions, $a = 0$, $b = 1$, and $r^2 = 1$.

also true of the Hénon attractor. To us, the phase portraits look as if they are distorted and superimposed views of the Hénon attractor.

For the geomagnetic reversal record we obtain apparent convergence at $D_C \sim 3$ (Figure 10b), but the scaling region is not clearly defined. Therefore as explained above, this result is not quantitatively meaningful. In contrast, D_C does not converge for any of the randomized data records but increases indefinitely with the embedding dimension D_e (Figure 10c). Similar results are also obtained when this analysis is made on the geomagnetic reversal timescales of *Harland et al.* [1982] and of *Kent and Gradstein* [1986]. In a further test we found that random addition of 20–25 very short magnetic intervals to the Earth's magnetic field data sets results in apparent convergence of the correlation integrals to higher values of D_C (~4). These results do not let us assess quantitatively low-dimensional chaos in the geomagnetic reversal data. The Grassberger-Procaccia method shows a clear difference between measured and randomized reversal sets, which suggests that the dynamics of the geomagnetic field is not random. It may be low dimensional, but a Grassberger-Procaccia analysis of the present reversal data sets lets us only establish a lower limit for its correlation dimension: $D_C > 3$.

Summary and Conclusions

We have studied the record of reversals of the Earth's magnetic field from the present to 160 Ma using three techniques of nonlinear dynamics: phase portraits, nonlinear forecasting, and the Grassberger-Procaccia method. The analyzed geomagnetic reversal sequences appear to be unpredictable by nonlinear forecasting. The Grassberger-Procaccia method appears to converge to a low value ($D_C \sim$ 3) of the correlation dimension for the magnetic period sequence. Because of the small size of the geomagnetic reversal data set (only 282 points), and of the poor definition of the scaling region, this result is quantitatively not meaningful.

The clear difference in the correlation integrals between measured and randomized reversal sequences suggests that the geomagnetic reversal dynamics is not random and that low-dimensional chaos, if not detected, can be suspected.

We have produced synthetic reversal records from two self-excited bistable disk dynamo models [*Rikitake*, 1958; *Chillingworth and Holmes*, 1980] and have analyzed these records as we have analyzed the actual geomagnetic record. Phase portraits of the synthetic reversal records reveal a

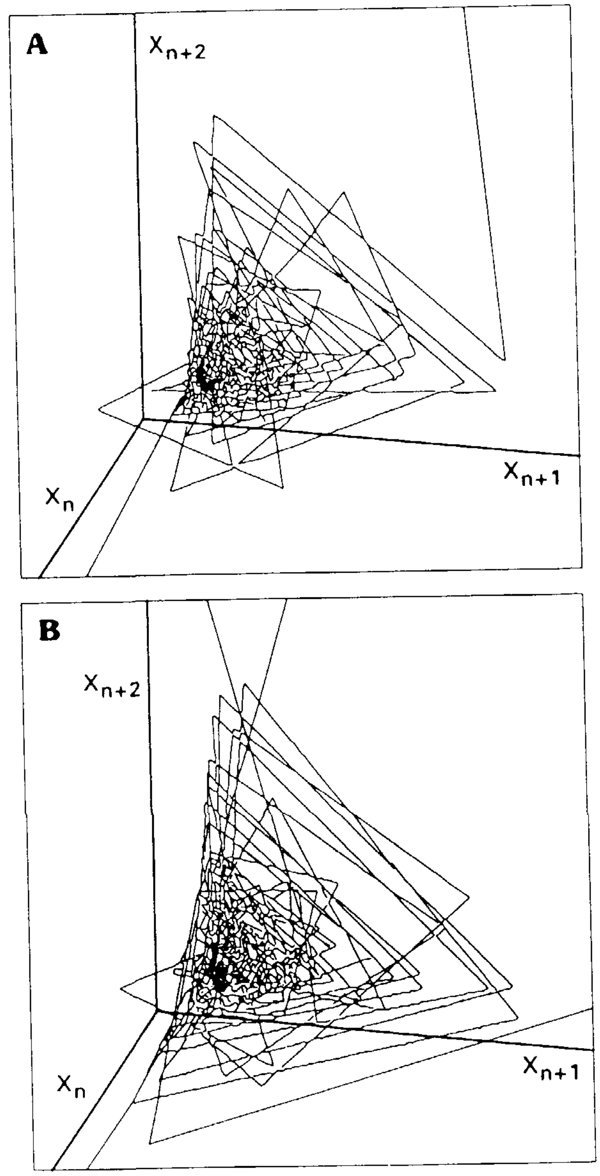


Figure 9. A three-dimensional time lag phase portrait for the Earth's geomagnetic field (EGF) reversal data sequence (the representative point of the Cretaceous superchron is not plotted). (a) EGF reversal data. (b) Same data as in Figure 4a, after randomization.

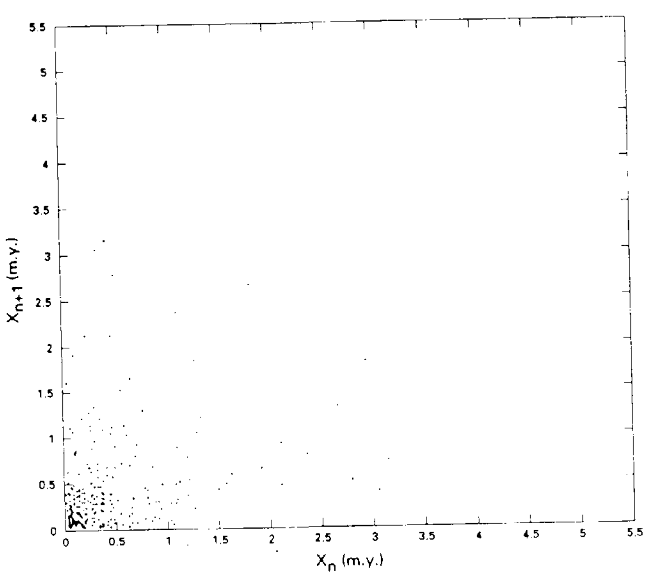


Figure 8. Two-dimensional time lag phase portrait for the Earth's magnetic field reversal data sequence X_n. Although this phase portrait appears featureless, there is some resemblance between the density distribution of the points in this figure and those in Figures 4 or 7a. The representative point of the Cretaceous superchron (which lasted about 35 m.y.) is not plotted.

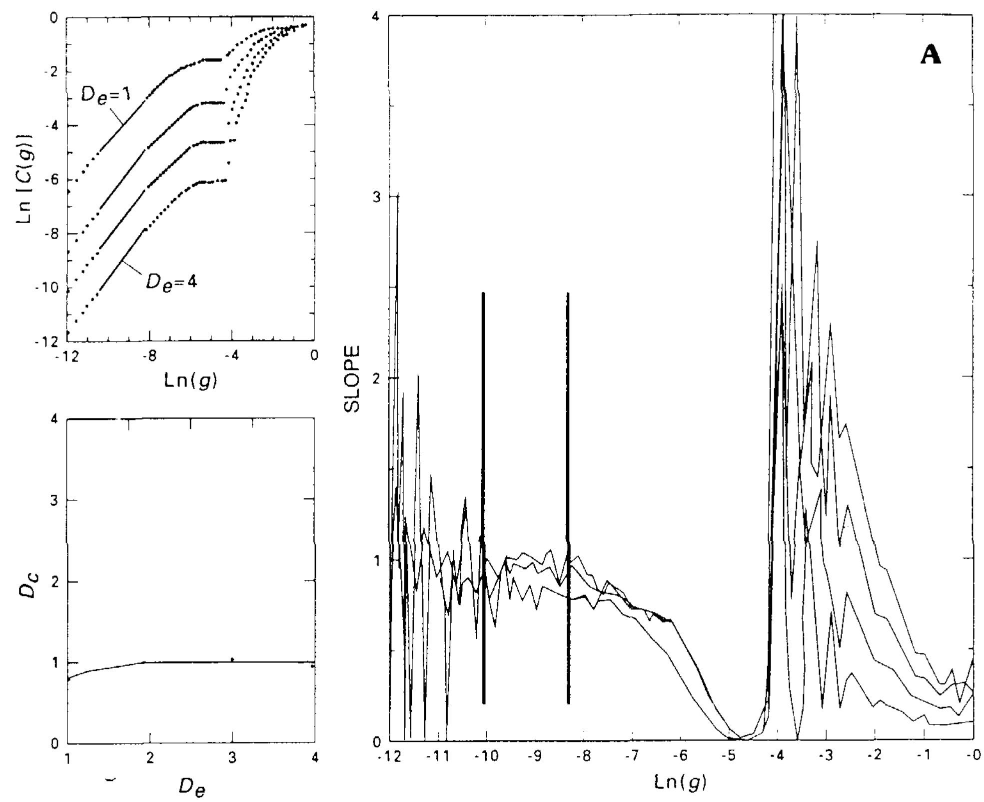


Figure 10. An evaluation of the correlation dimension D_C of the dynamo model and the geomagnetic reversal data. The left upper window shows correlation integrals $C(g)$ plotted against normalized length scales g for different embedding dimensions of the time series, on a ln-ln scale. Solid lines show intervals used to calculate the slopes. The slopes yield the correlation dimensions D_C, which are plotted against the embedding dimensions D_e in the bottom left window. The error bars in the lower left window represent the analytical errors on the slope regression. The large window on the right shows the slopes of the correlation integrals as a function of the natural logarithm of the correlation length g; the heavy vertical lines show the intervals used to calculate the slopes. (a) Data sequence produced by integrating (1) (Rikitake model dynamo; parameter values are $k = 2$, $\mu = 1.5$, $dt = 0.0005$; 4007 points). Note that the slopes of the correlation integrals converge to about one for low values of $\ln(g)$; see text. (b) Geomagnetic reversal data. (c) Same data as in Figure 10b, after randomization.

diversity of complex structures, apparently fractal, that vary widely as a function of the model parameters. Structures with similar characters appear in both model dynamo systems and in the Lorenz system as well (Figures 4–7). Nonlinear forecasting can predict the synthetic reversal sequences quite well when the phase portraits are relatively simple. When the phase portraits are more complex, and the data set is reduced to 300 points, the predicting power of nonlinear forecasting drops to zero. This implies that the negative result obtained by nonlinear forecasting for the geomagnetic data does not rule out low-dimensional chaos for the Earth's magnetic field dynamics.

The correlation dimension D_C of the model reversal sequences is approximately one. Because the synthetic reversal records are longer (600–20,000 points) than the geomagnetic record, and because we consistently obtain $D_C \sim 1$, we feel that this result is quantitatively significant.

Low-dimensional chaos can appear in natural systems for two essentially different reasons. On one hand, as discussed in the introduction, a natural system can undergo self-organization. At the onset of convection, for example, the state space collapses onto itself, and the number of state variables can be reduced by orders of magnitude. On the other hand, low-dimensional chaos can appear in turbulent systems, like the weather, which are very high dimensional. For example, *Tsonis and Elsner* [1988] and *Essex et al.* [1987] examined the weather system at time scales of seconds and days, respectively, and found low values of D_C [see *Lorenz*, 1991, and references therein]. *Lorenz* [1991] proposed an elegant explanation for this apparent contradiction. He studied an artificially constructed high-dimensional system and showed that the Grassberger-Procaccia method may converge to low estimates of D_C if the analyzed variable is strongly coupled to only a few of the state variables of the system and loosely coupled to many others. His work explains why low-dimensional chaos may appear in highly complex systems exhibiting many degrees of freedom like the weather, especially when one is limited to exploration of particular spatial domains or timescales.

Thus in the case of the Earth's magnetic field, assessing low-dimensional chaos would not be enough. More thought, and better data, are needed to assess self-organization in the system of Figure 1.

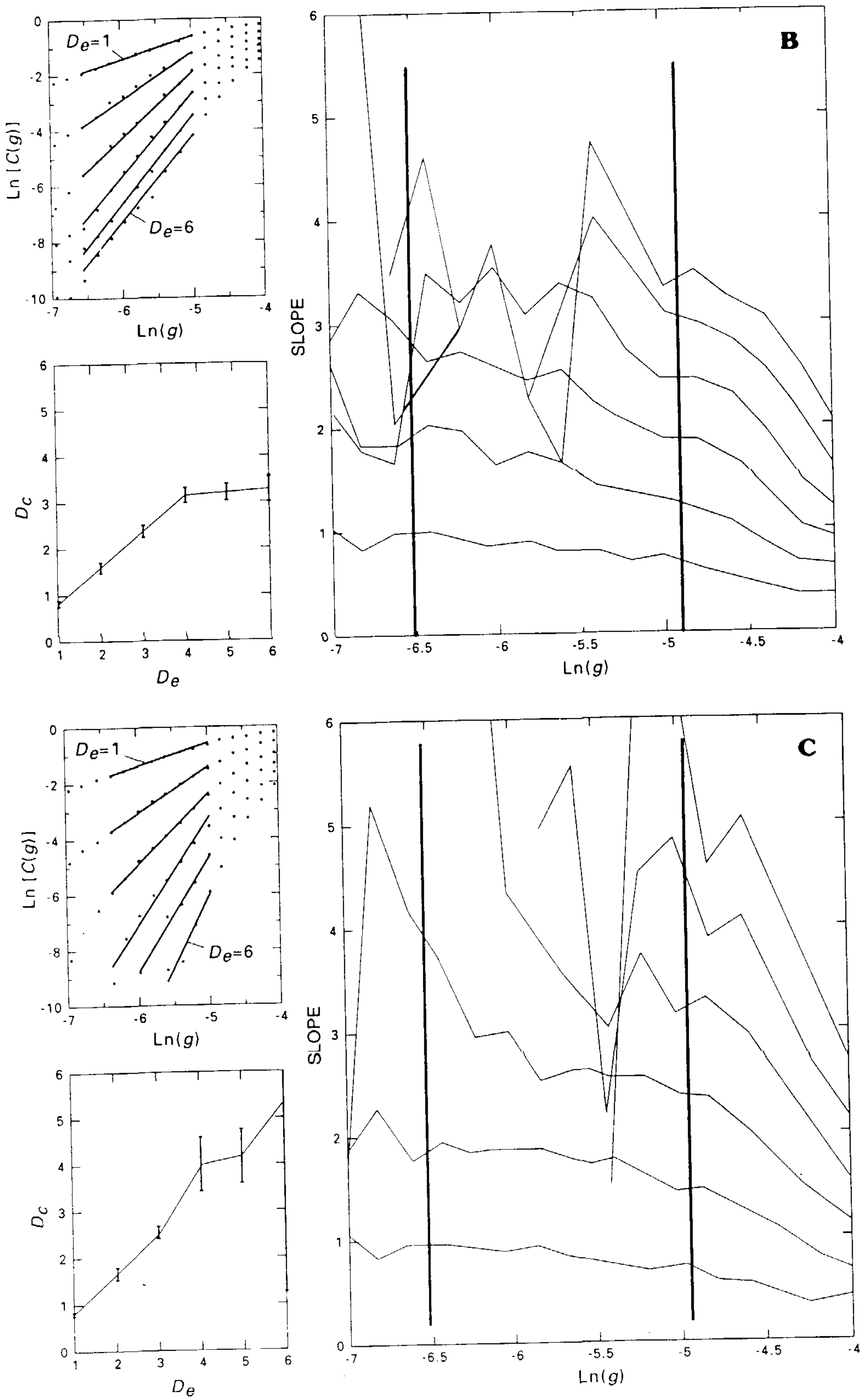


Figure 10. (continued)

Acknowledgments. Early versions of this paper benefited from review by USGS colleagues R. R. Charpentier and M. R. Karlinger, by A. Malinverno of Lamont-Doherty Geological Observatory of Columbia University, by E. N. Lorenz of MIT, and by discussions in the USGS Fractals/Chaos Study Group in Denver, Colorado. This paper greatly benefited by very careful reviews by M. Kono, of Tokyo University, and K. Ito, of Kobe University. Any use of trade names is for descriptive purposes only and does not imply endorsement by the U.S. Geological Survey.

References

Bergé, P., Y. Pomeau, and C. Vidal, *Order Within chaos: Towards a Deterministic Approach to Turbulence*, John Wiley, New York, 1984.

Boudreau, A. E., Examples of patterns in igneous rocks, in *Chemical Instabilities, NATO ASI Ser., Ser. C, Math. Phys. Sci.*, vol. 120, edited by G. Nicolis and F. Baras, pp. 299–304, D. Reidel, Norwell, Mass., 1984.

Cande, S. C., and D. V. Kent, A new geomagnetic polarity time scale for the Late Cretaceous and Cenozoic, *J. Geophys. Res.*, *97*, 13,917–13,951, 1992.

Casdagli, M., Nonlinear prediction of chaotic time series, *Physica D Amsterdam*, *35*, 335–356, 1989.

Chillingworth, D. R. J., and P. J. Holmes, Dynamical systems and models for reversals of the Earth's magnetic field, *Math. Geol.*, *12*, 41–59, 1980.

Cook, A. E., and P. H. Roberts, The Rikitake two-disk dynamo system, *Proc. Cambridge Philos. Soc.*, *68*, 547–569, 1970.

Cortini, M., and C. C. Barton, Nonlinear forecasting analysis of inflation-deflation patterns of an active caldera (Campi Flegrei, Italy), *Geology*, *21*, 239–242, 1993.

Cortini, M., L. Cilento, and A. Rullo, Vertical ground movements in the Campi Flegrei caldera as a chaotic dynamic phenomenon, *J. Volcanol. Geotherm. Res.*, *48*, 103–114, 1991.

Crossley, D., O. Jensen, and J. Jacobs, The stochastic excitation of reversals in simple dynamos, *Phys. Earth Planet. Inter.*, *42*, 143–146, 1986.

Cvitanovic, P. (ed.), *Universality in Chaos*, Adam Hilger, Bristol, England, 1984.

Dawkins, R., *The Blind Watchmaker*, Norton, New York, 1987.

Dubois, J., and C. Pambrun, Etude de la distribution des inversions du champ magnétique terrestre entre—165 Ma et l'actuel (échelle de Cox), Recherche d'un attracteur dans le système dynamique qui les génère, *C. R. Acad. Sci., Ser. II*, *311*, 643–650, 1990.

Eckmann, J.-P., and D. Ruelle, Fundamental limitations for estimating dimensions and Lyapunov exponents in dynamical systems, *Physica D Amsterdam*, *56*, 185–187, 1992.

Eigen, M., and P. Schuster, *The Hypercycle*, Springer-Verlag, New York, 1979.

Essex, C., T. Lookman, and M. A. H. Nerenberg, The climate attractor over short timescales, *Nature*, *326*, 64–66, 1987.

Farmer, J. D., and J. J. Sidorowich, Can new approaches to nonlinear modeling improve economic forecasts?, in *The Economy as an Evolving Complex System*, SFI Studies in the science of complexity, edited by P. W. Anderson, K. J. Arrow, and D. Pines, pp. 99–115, Addison-Wesley, Reading, Mass., 1988.

Friis-Christensen, E., and K. Lassen, Length of the solar cycle: An indicator of solar activity closely associated with climate, *Science*, *254*, 698–700, 1991.

Glass, B. P., and P. A. Zwart, The Ivory Coast microtektite strewnfield: New data, *Earth Planet. Sci. Lett.*, *43*, 336–342, 1979.

Glass, B. P., M. B. Swincki, and P. A. Zwart, Australasian, Ivory Coast and North American tektite strewnfield: Size, mass and correlation with geomagnetic reversals and other earth events, *Proc. Lunar Planet. Sci. Conf., 10th*, 2535–2545, 1979.

Gleick, J., *Chaos: Making a New Science*, Heinemann, London, 1988.

Grassberger, P., Do climatic attractors exist?, *Nature*, *323*, 609–612, 1986.

Grassberger, P., and I. Procaccia, Measuring the strangeness of strange attractors, *Physica D Amsterdam*, *9*, 189–208, 1983.

Harland, W. B., A. V. Cox, P. G. Llevellyn, C. A. G. Pickton, A. G. Smith, and R. Walters, *A Geologic Time Scale*, Cambridge University Press, New York, 1982.

Hénon, M., A two-dimensional mapping with a strange attractor, *Commun. Math. Phys.*, *50*, 69–77, 1976.

Holden, A. V., and M. A. Muhamad, A graphical zoo of strange and peculiar attractors, in *Chaos*, edited by A. V. Holden, pp. 15–35, Princeton University Press, Princeton, N. J., 1986.

Hoshi, M., and M. Kono, Rikitake two-disk dynamo system: Statistical properties and growth of instabilities, *J. Geophys. Res.*, *93*, 11,643–11,654, 1988.

Ito, K., Chaos in the Rikitake two-disk dynamo system, *Earth Planet. Sci. Lett.*, *51*, 451–456, 1980.

Kent, D. V., and F. M. Gradstein, A Jurassic to recent chronology, in *The Geology of North America*, vol. M, *The Western North Atlantic Region*, edited by P. R. Vogt and B. E. Tucholke, pp. 45–50, Geological Society of America, Boulder, Colo., 1986.

Laj, C., A. Mazaud, R. Weeks, and M. Fuller, Geomagnetic reversal paths, *Nature*, *351*, 447, 1991.

Lorenz, E. N., Deterministic nonperiodic flow, *J. Atmos. Sci.*, *20*, 130–141, 1963.

Lorenz, E. N., Dimension of weather and climate attractors, *Nature*, *353*, 241–244, 1991.

Lowrie, W., Magnetic time scales and reversal frequency, in *Geomagnetism and Palaeomagnetism*, edited by F. J. Lowes et al., pp. 155–183, Kluwer Academic, Norwell, Mass., 1989.

Lutz, T. M., and G. S. Watson, Effects of long-term variation on the frequency spectrum of the geomagnetic reversal record, *Nature*, *334*, 240–242, 1988.

Marzocchi, W., and F. Mulargia, Statistical analysis of the geomagnetic reversal sequences, *Phys. Earth Planet. Inter.*, *61*, 149–164, 1990.

Marzocchi, W., and F. Mulargia, The periodicity of geomagnetic reversals, *Phys. Earth Planet. Inter.*, *73*, 222–228, 1992.

McBirney, A. R., and R. M. Noyes, Crystallization and layering of the Skaergaard intrusion, *J. Petrol.*, *20*, 487–554, 1979.

McFadden, P. L., and R. T. Merrill, Lower mantle convection and geomagnetism, *J. Geophys. Res.*, *89*, 3345–3362, 1984.

Merino, E., Survey of geochemical self-patterning phenomena, in *Chemical Instabilities, NATO ASI Ser., Ser. C, Math. Phys. Sci.*, vol. 120, edited by G. Nicolis and F. Baras, pp. 305–328, D. Reidel, Norwell, Mass., 1984.

Merrill, R. T., and M. W. McElhinny, *The Earth's Magnetic Field—Its History, Origin and Planetary Prospective*, Academic, San Diego, Calif., 1983.

Merrill, R. T., and P. L. McFadden, Paleomagnetism and the nature of the geodynamo, *Science*, *248*, 345–350, 1990.

Morner, N.-A., Changes in the Earth's rate of rotation on an El Nino to century basis, in *Geomagnetism and Palaeomagnetism*, edited by F. J. Lowes et al., pp. 45–53, Kluwer Academic, Norwell, Mass., 1989.

Nicolis, G., and I. Prigogine, *Self-Organization in Nonequilibrium Systems*, John Wiley, New York, 1977.

Prigogine, I., and I. Stengers, *Order out of Chaos*, Bantam, New York, 1984.

Rasband, S. N., *Chaotic Dynamics of Nonlinear Systems*, John Wiley, New York, 1990.

Ribes, E., J. C. Ribes, and R. Barthalot, Evidence for a larger Sun with a slower rotation during the seventeenth century, *Nature*, *326*, 52–55, 1987.

Rikitake, T., Oscillations of a system of disk dynamos, *Proc. Cambridge Philos. Soc.*, *54*, 89–105, 1958.

Robbins, K. A., A moment equation description of magnetic reversals in the earth, *Proc. Natl. Acad. Sci. U. S. A.*, *73*, 4297–4301, 1976.

Rochester, M. G., Causes of fluctuations in the rotation of the Earth, *Philos. Trans. R. Soc. London A*, *313*, 95–105, 1984.

Sagdeev, R. Z., D. A. Usikov, and G. M. Zaslavsky, *Nonlinear Physics—From the Pendulum to Turbulence and Chaos*, Harwood, Chur, Switzerland, 1988.

Schaffer, W. M., and C. W. Tidd, *Nonlinear Forecasting for Dynamical Systems*, Dynamical Systems, Tucson, Ariz., 1990.

Schaffer, W. M., G. L. Truty, and S. L. Fulmer, *Dynamical Software—User's Manual and Introduction to Chaotic Systems*, Dynamical Systems, Tucson, Ariz., 1988.

Shaw, R., *The Dripping Faucet as a Model Chaotic System*, Aerial, Santa Cruz, Calif., 1984.

Shimizu, M., and Y. Honkura, Statistical nature of polarity reversals of the magnetic field in coupled-disk dynamo models, *J. Geomagn. Geoelectr.*, *37*, 455–497, 1985.

Sornette, A., J. Dubois, J. L. Cheminée, and D. Sornette, Are

sequences of volcanic eruptions deterministically chaotic?, *J. Geophys. Res.*, *96*, 11,931–11,945, 1991.
Stewart, C. A., and D. L. Turcotte, The route to chaos in thermal convection at infinite Prandtl number, 1, Some trajectories and bifurcations, *J. Geophys. Res.*, *94*, 13,707–13,717, 1989.
Sugihara, G., and R. M. May, Nonlinear forecasting as a way of distinguishing chaos from measurement error in time series, *Nature*, *344*, 734–741, 1990.
Takens, S., Detecting strange attractors in turbulence, in *Dynamical Systems and Turbulence*, edited by D. A. Rand and L. S. Young, pp. 366–381, Springer-Verlag, New York, 1981.
Thompson, J. M. T., and H. B. Stewart, *Nonlinear Dynamics and Chaos*, John Wiley, New York, 1986.
Tritton, D. J., Deterministic chaos, geomagnetic reversals, and the spherical pendulum, in *Geomagnetism and Palaeomagnetism*, edited by F. J. Lowes et al., pp. 215–226, Kluwer Academic, Norwell, Mass., 1989.
Tsonis, A. A., and J. B. Elsner, The weather attractor over very short time scales, *Nature*, *333*, 545–547, 1988.
Tsonis, A. A., and J. B. Elsner, Comments on "Dimension analysis of climatic data", *J. Clim.*, *3*, 1502–1505, 1990.
Vogt, P. R., Changes in geomagnetic reversal frequency at times of tectonic change: Evidence for coupling between core and upper mantle processes, *Earth Planet. Sci. Lett.*, *25*, 313–321, 1975.
Wager, L. R., and G. M. Brown, *Layered Igneous Rocks*, Oliver and Boyd, Edinburgh, 1968.
Wolf, A., Quantifying chaos with Lyapunov exponents, in *Chaos*, edited by A. V. Holden, pp. 273–290, Princeton University Press, Princeton, N. J., 1986.

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(Received March 29, 1993; revised May 2, 1994; accepted May 10, 1994.)